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The role of initial shares in multi-period production economies with incomplete markets *

Egbert Dierker †

January 28, 2017

Abstract

This paper focuses on a single firm with constant returns to scale in a multi-period setting with incomplete markets and a single good per state. Profits vanish whenever the firm maximizes profits with respect to a given price system. The paper addresses the following question: Shall the firm always act as a price taker? In the case of a partnership, there are no initial shares and no profits accrue from production. A corporation, however, has initial shareholders and can sell its output at any price. An example shows that this additional freedom can improve efficiency and welfare. This results from the fact that a wedge between price and cost can mitigate the inefficiency caused by the consumers who disregard the impact of their initial portfolio decisions on subsequent markets.

Keywords: Multi-period economies with incomplete markets, partnerships and corporations, competitive price perceptions, the role of initial shares, the objective of a firm, efficiency and social welfare

JEL Classification: D21, D52, D61

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1 Introduction.

This paper focuses on a single firm with constant returns to scale in a setting with incomplete markets, more than 2 time periods and a single good per state. Whenever the firm maximizes profits with respect to a given price system, the price of the output equals its production cost. The paper addresses the following question: *Shall the firm always act as a price taker or is it possible to improve efficiency and welfare by selling the output above or below costs?*

This question is studied from a purely normative perspective in a particularly simple and transparent model. The only assets are shares in the firm. The firm faces no competition and there is no strategic interaction.

Two different types of firms, partnerships and corporations, are compared; see §31 and §32 of Magill and Quinzii (1996), henceforth referred to as MQ. The main difference between the types is that a partnership has no initial owners whereas a corporation is initially owned by consumers. In the case of a corporation, the initial shares $\delta^i \geq 0$ with $\sum_i \delta^i = 1$ are given exogenously and traded at $t = 0$.

In the case of a *partnership*, a group of consumers gets together to found a firm. Because of constant returns to scale, there are no incentives to exclude a consumer. Suppose the partnership chooses the production plan $y = (y_0, y_+) \in \mathbb{R}_- \times \mathbb{R}^S_+$, where $|y_0| = C$ denotes the cost to be paid at $t = 0$ and y_+ the stochastic dividend stream accruing at S future states. Each partner i chooses his share ϑ^i_0 of the production plan. In equilibrium, $\sum_i \vartheta^i_0 = 1$.

By definition, a partnership provides its output y_+ in exchange for the production cost. Thus, the above question can be reformulated as follows. *Shall firms be organized as partnerships?* When there are only two periods, partnerships have a solid theoretical foundation as shown in the seminal article by Drèze (1974). In this case, the shares ϑ^i_0 are the final shares and the firm should maximize profits with respect to the price system $\pi = \sum_i \pi^i \vartheta^i_0$ where π^i is consumer i's state price system (or vector of stochastic discount factors).

In the case of a *corporation*, consumer i is endowed with the initial share $\delta^i \geq 0$ where $\sum_i \delta^i = 1$. That is to say, i owns the share δ^i of the firm's production plan $y = (y_0, y_+)$. This obliges i to pay $\delta^i|y_0|$ at $t = 0$. Furthermore, δ^i entitles i to receive $\delta^i y_+$. This right can be sold on the stock market at $t = 0$ at the market clearing price q_0. Trading at $t = 0$ converts the initial shares δ into intermediate shares denoted ϑ_0 which are carried over to the next period. Consumer i's net payment to obtain the share $\vartheta^i_0 \geq 0$ is $(\vartheta^i_0 - \delta^i)q_0$. In equilibrium, q_0 is such that $\sum_i \vartheta^i_0 = \sum_i \delta^i = 1$. The price q_0

can be lower or higher than the production cost C. The definition of a corporation is silent about the relationship between production cost and output value.

There is a stock market at any non-terminal state s. Shares carried over to s are traded at price q_s. Let ϑ_s^i denote i's shares carried over from s to one of its successors. In a *stock market equilibrium*, $\sum_i \vartheta_s^i = 1$ for every non-terminal s.

Apart from $t = 0$, there is no difference between the description of a partnership or a corporation. Loosely speaking, a partnership is a corporation with constant returns to scale, a missing stock market at $t = 0$, and price taking behavior. Can the richer framework of a corporation provide socially desirable opportunities?

When one wants to convert a corporation with constant returns to scale into a partnership one has to abolish the initial shares δ^i. This can be done by imposing the pricing rule $q_0 = C$, which is a special case of the marginal cost pricing rule. In a corporation, i's consumption at $t = 0$ is $x_0^i = e_0^i - \delta^i C + (\delta^i - \vartheta_0^i)q_0 = e_0^i + \delta^i(q_0 - C) - \vartheta_0^i q_0$ where e_0^i is i's initial endowment at $t = 0$. If $q_0 = C$ the initial shares δ^i vanish so that $x_0^i = e_0^i - \vartheta_0^i C$ as in the partnership.

It is instructive to consider the case in which, for every consumer i, the initial shares δ^i coincide with the shares ϑ_0^i deliberately chosen by i at $t = 0$. Then i's demand $x_0^i = e_0^i - \delta^i C + (\delta^i - \vartheta_0^i)q_0$ for good 0 in the case of a corporation coincides with i's demand $x_0^i = e_0^i - \vartheta_0^i C$ in the case of a partnership for every i at $t = 0$. However, unless i's utility is quasilinear, δ^i will typically impact i's demand for shares at subsequent stock markets. Therefore, the original shares δ^i typically create long lasting market repercussions although the individual shareholdings remain unchanged during the initial period.

Social welfare maximization takes into account how the original shares δ^i impact market outcomes. When the initial shares are sold below costs the net sellers of initial shares subsidize the net buyers. When the shares are sold above cost the redistribution of wealth is reversed. In a partnership, all market transactions leave the distribution of wealth unaltered.

The objective of a firm used in this paper can be described most easily in the case of a corporation. Assume for simplicity that every consumer holds at least a tiny amount of initial shares so that the welfare of the initial owners coincides with the welfare of the society. This assumption rules out that the group of initial owners exploits the rest of the economy. The corporation chooses, as in a Cournot model, an output vector y_+. All functions used to analyze the model depend directly or indirectly on y_+. When the production plan y has been chosen, consumer i possesses the intermediate endowment

$e^i + \delta^i y$ where $e^i \in \mathbb{R}_+^{(S+1)}$ is i's initial endowment. Consumers anticipate the market clearing prices correctly and determine their optimal trades on all markets. In equilibrium, all markets clear.

Every utility function is normalized such that the marginal utility of good 0 equals 1 at the optimum. The (indirect) social welfare function $\mathcal{W}(y_+)$ is the sum of all normalized indirect utility functions. *The corporation chooses its production such that the first order condition $D\mathcal{W}(y_+) = 0$ for welfare maximization is satisfied.* For a more extensive explanation, see Section 2.

In the case of a partnership, the basic principle is the same. However, the firm takes the constraint $q_0 = C$ into account. *The partnership aims to satisfy the first order condition for constrained welfare maximization.* It is worth emphasizing that the degree of complexity of multi-period models of production economies with incomplete markets in the Walrasian tradition comes close to that of models with Cournot competition.

1.1 Relationship to the literature

Gabszewicz and Vial (1972) introduce a model that combines Cournot-Nash competition with Walrasian exchange of consumption goods under the assumption that markets are complete. The basic idea can be described as follows. The consumption goods are produced by firms who need non-marketable primary factors as inputs. Every firm chooses its production plan. The consumers possess preassigned shares of the firms, provide the primary factors in accordance with their shares, and receive their shares of the firms' output. Thereafter, Walrasian exchange of the consumption goods takes place at market clearing prices. The main difference between Gabszewicz and Vial (1972) and the present paper is that they focus on oligopolistic competition whereas this paper focuses on market incompleteness.

Both papers have in common that they deal with preassigned, initial shares. First, the production plans are chosen. Thereafter, the output is distributed and the consumers obtain their intermediate endowments. Finally, Walrasian exchange takes place and the intermediate endowments are traded at their equilibrium prices. In multi-period models of corporations, Walrasian exchange occurs repeatedly. Both papers deal with the redistribution of initial wealth, however, from different perspectives. Gabszewicz and Vial focus on the profit motive of oligopolists whereas this papers abstracts from that motive and uses redistribution in order to enhance efficiency and welfare.

Guesnerie (1975) points out that a redistribution of wealth can be needed in order to achieve a Pareto improvement when one leaves the classical Arrow-Debreu framework. In his paper, the aggregate production set fails to be convex and marginal cost pricing becomes a necessary requirement for Pareto efficiency. Several marginal cost pricing equilibria exist, however, none of them is Pareto efficient given the distribution of the firms' profits or losses. In order to obtain a Pareto efficient marginal cost pricing equilibrium, the original distribution scheme needs to be changed. According to the fundamental theorems of welfare economics, no such problem arises in the convex case.

The situation is similar when preferences and technologies are convex and markets are incomplete. Section 3 of this paper presents an example of an economy with three types of consumers. One of the types has a quasilinear utility function. When all initial shares are held by the quasilinear type, the initial shares δ^i do not impact the demand for the intermediate shares ϑ_0^i due to the absence of income effects. Otherwise, the income effects impact, directly or indirectly, all market clearing prices. The more initial shares are held by non-quasilinear consumers, the larger is the potential impact of initial shares on stock market prices. By driving a wedge between q_0 and C, initial shares can impact all market clearing prices. The introduction of initial shares resembles the introduction of a tax in an economy with distorted markets. *Initial shares provide corporations with a tool that can help to reduce existing distortions.* This tool is not available in a multi-period partnership.

There is a long tradition to assume competitive behavior in general equilibrium models with or without incomplete markets. Grossman and Hart (1979) use competitive price perceptions in two- as well as multi-period models with corporations. In their paper, a corporation maximizes profits with respect to a convex combinations of utility gradients where the weights are the initial shares δ^i. This paper, however, makes the point, that *inefficiencies caused by consumption decisions in a multi-period setting can be mitigated by corporations provided that there are no competitive price perceptions and all market interactions are correctly taken into account.*[1]

Multi-period partnerships have been analyzed in Dierker (2015) in order to generalize the original Drèze rule. The generalized Drèze rule is much more complex than the classical Drèze rule in the two-period case. In particular, it takes all individual transactions into account and the price system can no longer be expressed in terms of utility gradients. The difference between the two- and the multi-period case is due to the fact that the envelope theorem can no longer be applied when there are more than two periods. The treatment of multi-period partnerships in this paper follows Dierker (2015).

[1] Competitive price perceptions can entail unintended welfare losses in multi-period models because beneficial redistribution of wealth caused by initial shares are ruled out.

2 Model and conceptual background.

2.1 Corporations, partnerships, and their objectives.

It suffices to consider a three-period economy whose underlying date-event tree has the initial state $s = 0$ at $t = 0$ and states $s = 1, \ldots, S$ at $t > 0$. There is a single good per state and a single firm with constant returns to scale technology $\mathcal{Y} \subset \mathbb{R}_- \times \mathbb{R}^S_+$. The firm can be a corporation or a partnership.

To define social welfare in either case, every (indirect) utility function is normalized such that the marginal utility of good 0 equals 1 at the equilibrium allocation under consideration. That is to say, if one additional marginal unit of good 0 becomes available at the reference equilibrium, social welfare increases by one unit independently of who consumes the marginal unit of good 0. Consumer i's normalized utility gradient π^i describes i's state price system or vector of stochastic discount factors. The social welfare of a group of consumers is the sum of the normalized indirect utility functions of its members. This paper focuses on the social welfare of all consumers.

Consider first the case of a *corporation*. There is a stock market at each non-terminal node. The implicit function theorem is used to express all functions directly or indirectly as functions of y_+. First one determines, for every consumer i and every non-terminal node s, the demand $\vartheta^i_s(y_+)$ for shares which determine i's consumption $x^i(y_+)$. Then one solves the system of market clearing equations to obtain an equilibrium price vector.

In a corporation economy, the output y_+ is sold at the market clearing price $q_0(y_+)$. The set of stock market equilibria is characterized by

$$\mathcal{Y}\text{corp} = \{y_+ \subset \text{proj}_2\mathcal{Y} \mid \sum_i \vartheta^i_s(y_+) = 1 \text{ for every non-terminal state } s\},$$

where proj_2 denotes the projection to $\mathbb{R}^S_+$.

Assume that there is a planner who can choose the production plan and make infinitesimal transfers of good 0.[2] Can the planner find a first order Pareto improvement over the allocation of the reference equilibrium induced by y^*_+? To answer this question, define *social welfare* as

$$\mathcal{W}_{y^*}(y_+) = \sum_i \frac{U^i(x^i(y_+))}{\partial_0 U^i(x^i(y^*_+))}. \tag{1}$$

[2]A planner associated with constrained efficiency is much stronger because he can also assign shares to consumers.

Whenever $D\mathcal{W}_{y^*}(y^*_+)$ does not vanish, a first order Pareto improvement exists. To avoid such equilibria, corporations are required to satisfy the first order condition $D\mathcal{W}_{y^*}(y^*_+) = 0$ for welfare maximization. When one differentiates $\mathcal{W}_{y^*}(y_+)$ with respect to $y_s, s = 1, \ldots, S$, one obtains, dropping the arguments, the first order condition

$$\partial_s y_0 + \sum_{i=1}^{I} \sum_{\sigma=1}^{S} \pi^i_\sigma \; \partial_s x^i_\sigma = 0 \text{ for } s = 1, \ldots, S. \tag{2}$$

The *objective of the corporation* is to satisfy condition (2). A stock market equilibrium is a *corporation equilibrium* iff $D\mathcal{W}_{y^*}(\hat{y}^*) = 0$. Observe that equation (2) is significantly more complex than a convex combination of utility gradients π^i. In contrast to the two-period case, $\pi^i_\sigma \; \partial_s x^i_\sigma$ does typically not vanish when $s \neq \sigma$.

In a two-period model, the condition $D\mathcal{W}_{y^*}(y^*_+) = 0$ characterizes Drèze equilibria. Originally, the concept of a Drèze equilibrium has been based on the first order condition for constrained efficiency [see Drèze (1974)]. This efficiency concept is no longer appropriate when there are at least three periods or multiple goods per state because exchange economies become generically constrained inefficient [see Geanakoplos et al. (1986)]. The efficiency concept used in this paper is much less demanding than constrained efficiency. It has been introduced under the name of minimal (constrained) efficiency in Dierker et al. (2005)) in a two-period model.

The planner associated with minimal efficiency cannot affect future consumption other than by choosing production plans. Thus, future consumption is constrained in the same way as in the case of welfare maximization. Consider the following procedure. At the first stage, the planner chooses production plans. At the second stage, the consumers, who have correct expectations of the production plans and the market clearing prices, choose their shares and determine their consumption plans. At the final stage, when all stock markets are closed so that shareholdings cannot be changed, the planner can redistribute the total consumption at $t = 0$. An allocation is minimally constrained efficient, or *minimally efficient* for short, if this planner cannot make a Pareto improvement. For a formal definition of a cardinal measure of minimal efficiency, see Subsection 2.3.

One may feel tempted to require the corporation to fulfill more than the first order condition for welfare maximization. However, the following problem arises already in the two period case. In that particular setting, the first order condition for welfare maximization coincides with the first order condition for constrained efficiency. E. and H. Dierker (2010b) consider two-period economies and present robust examples that show that a unique

Drèze equilibrium can maximize welfare although it is not minimally efficient. The Drèze equilibrium can also minimize welfare although it is constrained efficient. This can be explained as follows.

Subsection 2.2 introduces two Hicksian surplus concepts, the compensated and the equivalent surplus. The first one measures efficiency changes and the second one measures welfare changes. The difference between the two surplus concepts is of second order. More precisely, the critical points of the two surplus concepts coincide but the second derivatives at a critical point can have different signs. It is possible that one surplus function attains its maximum where the other surplus function attains its minimum. Therefore, *the goal of a firm is defined such that it does not discriminate between welfare and efficiency maximization.* This property is lost when one takes higher order effects into account.

Turn now to the case of *partnership* economies. At $t = 0$, every consumer i can become a partner by obtaining the share $\vartheta_0^i > 0$ of the output y_+ in exchange for the cost share $\vartheta_0^i C$. The partnership operates at a scale that is determined by the condition $\sum_i \vartheta_0^i = 1$.

At $t = 1$, the partnership goes public. There is a stock market at every non-terminal node $s \geq 1$ on which the shares $\vartheta_s^i(y_+)$ are sold at the market clearing price $q_s(y_+)$. In equilibrium, all stock markets clear, that is to say, $\sum_i \vartheta_s^i(y_+) = 1$. A *partnership equilibrium* is a stock market equilibrium with the property $D\mathcal{W}_{y^*}(\hat{y}^*) = 0$.

In the case of a partnership economy, the set of stock market equilibria is characterized by

$$\mathcal{Y}_{\text{part}} = \{y_+ \subset \text{proj}_2\mathcal{Y} \mid q_0(y_+) = C(y_+) \text{ and } \sum_i \vartheta_s^i(y_+) = 1 \text{ for all markets}\}.$$

One might think that a partnership is driven by efficiency reasons to satisfy $q_0 = C$. However, this reasoning is flawed. Price taking behavior entails inefficiencies in the consumption sector because every price taking consumer i ignores the impact to his choice of ϑ_0^i on subsequent market prices. This entails that the constraint $q_0 = C$ must be explicitly incorporated in the partnership's objective. Therefore, the equation $\sum_i \vartheta_0^i(y_+) = 1$ is used, together with the definition of $C(y_+)$, in order to express y_S as a function of $y_1, \ldots, y_{S-1}$.

In the two-period case, the envelope theorem applies. That is to say, the chain rule can be disregarded when one evaluates $D\mathcal{W}_{y^*}(y_+)$ and utility gradients together with the (final) shareholdings suffice to express the firm's goal. As a consequence, the firm pursues the goal to satisfy the first order condition for constrained efficiency that characterizes Drèze equilibria [see Section 6 of Dierker (2015)].

2.2 A cardinal efficiency measure, a cardinal welfare measure, and Kaldor-Hicks comparisons.

To shed light on the role of initial shares, a partnership equilibrium will be compared to a corporation equilibrium for a given assignment of initial shares. Call one of the equilibria A and the other one B. In many cases, no Pareto comparison exists, but one can still perform tests à la Kaldor-Hicks to compare A and B. Two tests will be applied, one is based on the compensating variation $CS_{y^*}(y_+)$, the other on the cardinal welfare measure introduced below. Both measures are expressed in units of good 0.

Consider two equilibria, a reference equilibrium associated with y_+^*, the output at the status quo, and some alternative equilibrium associated with the output y_+. Assume that the move to the alternative has been carried out and look backwards from y_+ to the status quo at y_+^*. Consumer i's *compensating surplus* $CS^i_{y^*}(y_+)$ is the amount of good 0 which i has to *lose* after the move from the reference stock market equilibrium to the alternative equilibrium; cf. Hicks (1956) and E. and H. Dierker (2010b). That is to say, $CS^i_{y^*}(y_+)$ compensates i for the move from y_+^* to y_+. Formally, $CS^i_{y^*}(y_+)$ is defined implicitly by

$$U^i(x_0^i(y_+ - CS^i_{y^*}(y_+),\ x_+^i(y_+)) = U^i(x^i(y_+^*)). \tag{3}$$

The *total compensating surplus* associated with the change from y_+^* to y_+ is

$$CS_{y^*}(y_+) = \sum_i CS^i_{y^*}(y_+). \tag{4}$$

The total compensating surplus $CS_{y^*}(y_+)$ is the amount of good 0 that can be taken out of the economy at y_+ without making any consumer worse off than at y_+^*. It is an indicator of the inefficiency of the status quo in comparison to the alternative. *The reference equilibrium is minimally efficient iff $CS_{y^*}(y_+) \leq CS_{y^*}(y_+^*) = 0$ for all available alternatives y_+.* Observe that the set of available alternatives depends on whether the firm is organized as a corporation or a partnership.

A calculation shows that one obtains the first order condition (2) that is used to define the goal of a firm when one differentiates $CS_{y^*}(y_+)$ partially with respect to s using (3). Thus, *the first order condition for welfare maximization coincides with the first order condition for minimal efficiency.*

The definition of social welfare in (1) may appear puzzling for the following reason. $\mathcal{W}_{y^*}$ is defined as the sum of normalized utility functions that need not be cardinal. However, utilitarian welfare maximization relies on the interpersonal comparison of cardinal utility units. Therefore, $\mathcal{W}_{y^*}$ seems to look like a utilitarian utility function although it is not. Only the utility gradients are normalized. This suffices to define the goal of a firm because condition (2) depends only on the interpersonal comparison of marginal utility changes. The following surplus concept is based on cardinal unit comparisons and leads to utilitarian welfare functions.

Assume now that the move from the status quo at y_+^* to the alternative equilibrium at y_+ has not been made. Consumer i's *equivalent surplus* $ES_{y^*}^i(y_+)$ is the amount of good 0 which i has to *gain* at y_+^* in order to be indifferent to the move to y_+. Formally,

$$U^i(x_0^i(y_+^*) + ES_{y^*}^i(y_+),\ x_+^i(y_+^*)) = U^i(x^i(y_+). \tag{5}$$

The *total equivalent surplus* associated with the change from y^* to y_+ is

$$ES_{y^*}(y_+) = \sum_i ES_{y^*}^i(y_+). \tag{6}$$

Every alternative output plan y_+ is evaluated with the same function ES_{y^*}. This function is a utilitarian social welfare function because it measures, for every consumer i, the utility in additional units of good 0 at the equilibrium allocation associated with y_+^*. The plan y_+^* maximizes social welfare iff $ES_{y^*}(y_+) \leq ES_{y^*}(y_+^*) = 0$ for all available alternatives y_+. In the case of a partnership, prices $q_0 \neq C$ are not available whereas they are in the case of a corporation. Formulae (5) and (6) will be used to measure the welfare change caused by a move from y_+^* to y_+.

Suppose the economy is in equilibrium A and consider the social welfare function with the utility normalization made at A. A move from A to B causes a *welfare loss* if this welfare function assigns a lower value to B. That is to say, one needs to distribute less than the total initial endowment available at A in order to generate the utility profile attained at B.

Observe that a move from A to B causes an efficiency loss if and only if a move from B to A causes a welfare gain. In the quasilinear case, efficiency and welfare changes coincide. Otherwise, they may have different signs. As typical for surplus concepts, the first order conditions for the compensating and the equivalent surplus maximization coincide.

The issue of transitivity is avoided in this paper because only two equilibria at a time are compared. The joint use of both surplus concepts makes it possible to compare equilibria also in cases in which they cannot be Pareto ranked.

Consider a pair of economies that differ only by the existence or absence of original shares. It is shown that a partnership equilibrium can be Pareto dominated by a corporation equilibrium. Moreover, when a Pareto comparison is impossible the Kaldor-Hicks criterion can, to some extent, be used to conclude that a corporation equilibrium is socially preferable to a partnership equilibrium. The following proposition answers the question: *Shall a regulator always impose the condition $q_0 = C$ that characterizes a partnership?*

Proposition. *A corporation does always as well as a partnership when it imposes the constraint $q_0 = C$. A partnership equilibrium, however, can be Pareto dominated by a corporation equilibrium. When no Pareto comparison can be made, it is still possible that a move from a partnership equilibrium to a corporation equilibrium increases the total equivalent surplus as well as the total compensating surplus.*

3 Numerical example.

As shown in the introduction, consumer i's demand in the case of a partnership coincides with i's demand in the case of a corporation under the constraint $q_0 = 0$ because i's initial shares δ^i drop out. This section describes a numerical example of an economy that proves the essential part of the above Proposition.

The basic intuition behind the Proposition can be described as follows. Consider a partnership equilibrium with production plan y^* that maximizes CS_{y^*} as well as ES_{y^*}. Then $DCS_{y^*}(y^*_+) = DES_{y^*}(y^*_+)$ is orthogonal to the boundary of the set of all output plans y_+ that satisfy the constraint $q_0 = C$. Therefore, one can improve welfare and efficiency when this constraint is slightly relaxed. In the subsequent example, consumers of type Q have quasilinear utility functions. One can make the relaxation of $q_0 = C$ arbitrarily small by assigning all but arbitrarily few initial shares to consumers of type Q.

In the example, there are three time periods, $t = 0, 1, 2$, and seven states. State 0 at $t = 0$ is followed by states 1 and 2 at $t = 1$. At $t = 2$, states 3 and 4 follow state 1 and states 5 and 6 follow state 2. There is a single good at each state and a single firm. There are no securities other than shares in the firm.

Consider three types of consumers, A, B and Q, with additively separable, concave utility functions. The utility function of type Q is quasilinear. Define

$$\begin{aligned}
U^A(x_0, x_1, \ldots, x_6) &= 10\log(x_0) + 1\log(x_1) + 2\log(x_2) + 3\log(x_3)\\
&\qquad + 4\log(x_4) + 5\log(x_5) + 6\log(x_6),\\
U^B(x_0, x_1, \ldots, x_6) &= 10\log(x_0) + 3\log(x_1) + 2\log(x_2) + 1\log(x_3)\\
&\qquad + 1\log(x_4) + 2\log(x_5) + 3\log(x_6),\\
U^Q(x_0, x_1, \ldots, x_6) &= x_0 + \log(x_1) + \log(x_2) + \log(x_3)\\
&\qquad + \log(x_4) + \log(x_5) + \log(x_6).
\end{aligned} \tag{7}$$

respectively. For simplicity, there are no initial endowments except at $t = 0$ and every consumer is endowed with $e_0^A = e_0^B = e_0^Q = 30$. There are ten consumers of type A, ten of type B and fifty of type Q. A production plan is denoted $y = (y_0, y_+) \in \mathbb{R}_- \times \mathbb{R}_+^6$ where $y_+ = (y_1, \ldots, y_6)$. The cost $|y_0| = C(y_+)$ is

$$C(y_1, \ldots, y_6) = y_1 + y_2 + \ldots, y_6. \tag{8}$$

The example has been chosen such that that a move from the partnership equilibrium to a corporation equilibrium increases the total equivalent and the total compensating surplus also when type Q holds few or even no initial shares provided that types A and B possess similar amounts of initial shares.

Subsection 3.1 explains how the partnership equilibrium is computed and presents the numerical solution. Subsection 3.2 explains the computation of the corporation equilibrium. The relationship between the partnership equilibrium and the corporation equilibria, which are parameterized by the distribution of initial shares across types, is discussed in Section 4.

3.1 Partnership equilibrium.

In a partnership, consumer i consumes $e_0^i + \vartheta_0^i y_0$ at $t = 0$. The consumption at an intermediate node ξ_s is $x_s^i = q_s(\vartheta_{s_-}^i - \vartheta_s^i) + \vartheta_{s_-}^i y_s$ at $t = 1$, where ξ_{s_-} is the immediate predecessor of ξ_s. If ξ_s is a terminal node, then i consumes $x_s^i = \vartheta_{s_-}^i y_s$. The size of the partnership is such that the initial investments ϑ_0^i add up to 1.

The initial investment of a consumer of type A is $\vartheta_0^A = 630/(31\ C)$, $\vartheta_1^A = 2205(q_1 + y_1)/(124\ q_1\ C)$, and $\vartheta_2^A = 6930(q_2 + y_2)/(403\ q_2\ C)$ where the variable y_+ has been dropped. For consumers of type B, one obtains $\vartheta_0^B = 180/(11\ C)$, $\vartheta_1^B = 72(q_1 + q_2)/(11\ q_1\ C)$, and $\vartheta_2^B = 900(q_2 + y_2)/(77\ y_2\ C)$. A consumer of type Q demands $\vartheta_0^Q = 6/C$, $\vartheta_1^Q = 4(q_1 + q_2)/(q_1\ C)$, and $\vartheta_2^Q = 4(q_2 + y_2)/(q_2\ C)$. When the shares ϑ_0^i add up to 1 then $C = 227400/341 \approx$

666.862. Solving the market clearing equations for markets 1 and 2 leads to $q_1 = 60463/30497\ y_1 \approx 1.983\ y_1$ and $q_2 = 151693/55241\ y_2 \approx 2.746\ y_2$.

Let $\hat{y} = (y_1, \ldots, y_5)$. The cost function (8) is used to eliminate the last component y_6 of y_+ by defining

$$y_6 = g(\hat{y}) = 227400/341 - y_1 - \cdots - y_5. \tag{9}$$

The fact that C is constant results from the Cobb-Douglas nature of the preferences for y_+. In general, the implicit function theorem is used to define $g(\hat{y})$. Equation (9) provides an equilibrium condition that must be taken into account in a multi-period model.

Dropping the variable $\hat{y}$, i's consumption equals

$$x^i = (e^i_0 - \vartheta^i_0 C,\ q_1(\vartheta^i_0 - \vartheta^i_1) + \vartheta^i_0 y_1,\ q_2(\vartheta^i_0 - \vartheta^i_2) + \vartheta^i_0 y_2,\ \vartheta^i_1 y_3,\ \vartheta^i_1 y_4,\ \vartheta^i_2 y_5,\ \vartheta^i_2 g).$$

Let $u^i(\hat{y}) = U^i(x^i(\hat{y}))$ be the utility i obtains when $\hat{y}$ is chosen.

All consumers are partners so that the firm acts on behalf of the whole society. Because $x^A_0 = 30 - 630/31 = 300/31$, A's marginal utility of good 0 equals 31/30. Similarly, B's marginal utility equals 11/15. Thus, both normalization factors, $nf^A = 30/31$ and $nf^B = 15/11$, are independent of the allocation. In the case of the quasilinear consumer Q, no normalization is needed. Since there are 10 consumers of type A, 10 of type B, and 50 of type Q social welfare is given by

$$\mathcal{W}(\hat{y}) = 10\ nf^A\ U^A(x^A(\hat{y})) + 10\ nf^B\ U^B(x^B(\hat{y})) + 50\ U^Q(x^Q(\hat{y})). \tag{10}$$

The first order condition $D\mathcal{W}(\hat{y}) = 0$ can be solved algebraically. For simplicity, numerical approximations are used to replace fractions and one obtains $\hat{y}^* \approx (100.587,\ 96.6276, 92.6686,\ 102.346,\ 125.66)$. The cost is $C = 227400/341 \approx 666.862$ and the last coordinate of the production plan y^* is $y^*_6 = C - (y_1 + \cdots + y_5) \approx 148.974$. The stock market prices are $q_1 \approx 199.422$ and $q_2 \approx 265.341$.

A consumer of type A, B, Q consumes, respectively,

$$x^A(\hat{y}^*) \approx (9.67742,\ 1.14284,\ 1.69707,\ 3.71744,\ 4.10566,\ 4.42033,\ 5.24044)$$
$$x^B(\hat{y}^*) \approx (13.6364,\ 4.41701,\ 2.53774,\ 1.36835,\ 1.51125,\ 3.00454,\ 3.56198)$$
$$x^Q(\hat{y}^*) \approx (24.0000,\ 0.89976,\ 1.08559,\ 0.83621,\ 0.92354,\ 1.02822,\ 1.21899).$$

The corresponding utility profile is approximately (50.8474, 39.1841, 23.9439). In Section 4, this profile is compared with the utility profiles of corporation equilibria for different allocations of initial shares.

3.2 Corporation equilibria.

Let δ^τ denote the amount of initial shares owned by an individual consumer of type $\tau = A, B, Q$. At $s = 0$, a consumer of type τ consumes the amount $x_0^\tau = 30 + \delta^\tau(q_0 - C) - \vartheta_0^\tau\, q_0$. The initial shares δ^τ change the consumption by $\Delta x_0^\tau = \delta^\tau(q_0 - C)$. When $\tau = A$ or $\tau = B$ there is an indirect impact on the demand for final shares caused by the income effect. This leads to

$$\vartheta_0^A = \frac{21(30 + \delta^A(q_0 - C))}{31 q_0}, \quad \vartheta_1^A = \vartheta_0^A\, \frac{7(q_1 + y_1)}{8 q_1}, \quad \vartheta_2^A = \vartheta_0^A\, \frac{11(q_2 + y_2)}{13 q_2} \tag{11}$$

and

$$\vartheta_0^B = \frac{6\ (30 + \delta^B(q_0 - C))}{11 q_0}, \quad \vartheta_1^B = \vartheta_0^B\, \frac{2(q_1 + y_1)}{5 q_1}, \quad \vartheta_2^B = \vartheta_0^B\, \frac{5(q_2 + y_2)}{7 q_2}. \tag{12}$$

Observe that, for $\tau = A, B$, the consumption change Δx_0^τ enters into the demand for intermediate shares ϑ_0^i and, because ϑ_1^A and ϑ_2^A are multiples of ϑ_0^A, also into the demand for final shares.

The quasilinear type Q is different because there is no income effect and

$$\vartheta_0^Q = \frac{6}{q_0}, \quad \vartheta_1^Q = \vartheta_0^Q\, \frac{2q_1 + y_1}{3 q_1}, \quad \vartheta_2^Q = \vartheta_0^Q\, \frac{2q_2 + y_2}{3 q_2}. \tag{13}$$

The initial shares δ^A and δ^B of the two non-quasilinear consumers impact all market clearing prices. The prices are

$$\begin{aligned}
q_0 &= \frac{30\ (77\ \delta^A\ C + 62\ \delta^B\ C - 7580)}{2310\ \delta^A\ + 1860\ \delta^B\ - 341} \\
q_1 &= \frac{105\ (77\ C - 12340)\ \delta^A\ +\ 24\ (124\ C + 54725)\ \delta^B - 604630}{105\ (11\ C + 12340)\ \delta^A\ +\ 24\ (186\ C - 54725)\ \delta^B - 304970}\ y_1 \\
q_2 &= \frac{21\ (847\ C - 75500)\ \delta^A\ +\ 30\ (403\ C + 7060)\ \delta^B - 1516930}{42\ (77\ C\ +\ 37750)\ \delta^A\ +\ 12\ (403\ C - 17650)\ \delta^B - 552410}\ y_2.
\end{aligned}$$

For $\tau = A, B$, the consumption change $\Delta x_0^\tau = \delta^\tau(q_0 - C)$ appears in the normalization factors of nf^τ of τ's utility function. These factors are equal to the equilibrium values of

$$nf^A = (30 + \delta^A(q_0 - C))/31 \quad \text{and} \quad nf^B = (30 + \delta^B(q_0 - C))/22,$$

respectively. Since the normalization factors are not constant they must be determined together with the optimal allocation. This completes the description of the *Cournot-Walras model of the corporation* apart from its objective.

Consider the welfare function

$$\mathcal{W}(y_+) = 10\, nf^A(y_+)\, U^A(x^A(y_+)) + 10\, nf^B(y_+)\, U^B(x^B(y_+)) + 50\, U^Q(x^Q(y_+)). \tag{14}$$

The main difference between (10) and (14) is that the welfare function in (14) depends on the S-dimensional output vector y_+ whereas it depends on the $(S-1)$-dimensional vector $\hat{y}$ in (10) due to the constraint $q_0 = C$.

When does q_0 equal C in the example? Because

$$q_0 - C = \frac{341\ C - 227400}{231\ \delta^A + 1860\ \delta^B - 341}$$

the price q_0 equals C if and only if $C = 227400/341$, Thus, *the constraint $q_0 = C$ is satisfied if and only if C is equal to the cost in the partnership equilibrium* of the previous subsection. This is the case if all original shares are owned by the quasilinear type Q.

4 The impact of initial shares on welfare and efficiency.

Let Δ^τ denote the aggregate amount of initial shares δ_i^τ held by all consumers of type τ where $\tau = A, B, Q$. Within a type, the initial shares are distributed equally. When $\Delta^Q = 1$, all initial shares are held by the quasilinear type. In this case, $q_0 = C$ and the corporation equilibrium coincides with the partnership equilibrium. This section contains a sequence of allocations of initial shares starting with small departures from the partnership equilibrium at $\Delta^Q = 1$.

Levels of Δ^A and Δ^B near 0.25 are of particular interest because they lead to utility profiles that are nearly proportional to those in the partnership equilibrium described in Subsection 3.1. This makes a Pareto comparison between the partnership equilibrium and the corporation equilibrium possible when Δ^A and Δ^B are suitably chosen. In various other cases, a comparison between the two equilibria can be made with the aid of the compensating and the equivalent surplus defined in Section 3.

In all cases, described here, the firm charges a price *below* the production cost C. The wedge increases when types A and B hold more initial shares. It is worth mentioning that one can easily find examples of economies in which q_0 lies above C. It can also happen that the signs of the total equivalent and of the total compensating surplus differ so that no comparison can be made.

4.1 The quasilinear type holds many initial shares.

The partnership equilibrium with production plan y^* is considered as the reference equilibrium or status quo. It coincides with the corporation equilibrium when $\Delta^Q = 1$. How does the cooperation equilibrium change when types A and B obtain more and more initial shares?

Assume that $\Delta^A = \Delta^B = 0.001$. Then $q_0/C \approx 1 - 10^{-6}$ and the output is strictly larger than in the partnership. In the corporation, the utility of all consumers at $t = 0$ is reduced while the utility at $t > 0$ is increased. A and B are better off in the corporation while Q is worse off. All three types gain from the expansion of the output. However, the cost of the expansion is essentially borne by type Q who sells initial shares below costs. *The partnership equilibrium is inefficient because price taking behavior leads to underproduction.* For the present parameter values, the size of the Hicksian surplus measures are $CS_{y^*} \approx -1.05 \times 10^{-9}$ and $ES_{y^*} \approx -3.8 \times 10^{-11}$.

For $\Delta^A = \Delta^B = 0.01$, the price cost ratio decreases to $q_0/C \approx 1 - 1.3 \times 10^{-5}$ and the output has increased further. Roughly speaking, types A and B hold now ten times more initial shares than before and the output increase is about ten times larger than in the previous case. Again, the bulk of the cost for this improvement is paid by type Q who subsidizes A and B. The surpluses become $CS_{y^*} \approx -1.05 \times 10^{-7}$ and $ES_{y^*} \approx -3.8 \times 10^{-9}$. As in the previous case, the society wants to move from the partnership equilibrium to the corporation equilibrium.

When $\Delta^A = \Delta^B = 0.1$ the wedge between q_0 and C increases further and $q_0/C \approx 1 - 1.5 \times 10^{-4}$. The output increase is nearly 10 times larger than in the case $\Delta^A = \Delta^B = 0.01$. As a consequence, the efficiency and welfare gains increase further. More precisely, $CS_{y^*} \approx -1 \times 10^{-5}$ and $ES_{y^*} \approx -2.9 \times 10^{6}$.

Until now changes in magnitude have occurred but otherwise there is little to report. Basically, type Q loses less than types A and B gain in total and Q's subsidies promote social welfare. The picture changes when one considers intermediate cases that are closer to the point where no trade in shares needs to be executed at $t = 0$.

4.2 More balanced distributions of initial shares.

Suppose $\Delta^Q = 0.5$ and $\Delta^A = \Delta^B = 0.25$. This case is of interest because the intermediate shares ϑ_0^i of each of the ten consumers i of type B are slightly below 0.025. Type Q does no longer subsidize type B but type A still gains from purchasing shares from Q at a price below costs. Only A is better off in the corporation. In equilibrium $q_0/C \approx 0.9995$, $CS_{y^*} \approx -5.3 \times 10^{-5}$ and $ES_{y^*} \approx -4.6 \times 10^{-5}$.

The next goal is to determine the production plan for which the amount of initial shares δ^i coincides with the amount ϑ_0^i of intermediate shares for every consumer i. In this case, no subsidization takes place because no trade occurs at $t = 0$. The shares of the three types are $\Delta^A \approx 0.3047$, $\Delta^B \approx 0.24536$, $\Delta^Q \approx 0.540$ and $q_0/C \approx 0.99952$.

A and B are worse off and Q is better off in the corporation. Furthermore, $CS_{y^*} \approx -5.06 \times 10^{-5}$ and $ES_{y^*} \approx -5.08 \times 10^{-5}$. The market at $t = 0$ is inactive but it impacts the prices in the subsequent markets. *The deliberate decision not to trade at* $t = 0$, which is impossible in a partnership, *enables the corporation to reach a better equilibrium.*

Can one find a Pareto improvement? To answer the question one perturbs the initial endowment such that the utility allocation of the corporation equilibrium is approximately proportional to that of the partnership equilibrium. It suffices to move from $\Delta^A \approx 0.3047$, $\Delta^B \approx 0.24536$ to $\Delta^A \approx 0.30347$, $\Delta^B \approx 0.24536$. Then $CS_{y^*} \approx -5.027 \times 10^{-5}$ and$ES_{y^*} \approx -5.026 \times 10^{-5}$ remain nearly unchanged. However, all individual surpluses become negative so that every consumer gains when the firm becomes a corporation. *The corporation equilibrium Pareto dominates the partnership equilibrium.*

4.3 Increasing the initial shares of types A and B further.

When $\Delta^A = \Delta^B = 0.3$ the situation is similar as in the case $\Delta^A = \Delta^B = 0.25$ considered above. This changes when $\Delta^A = \Delta^B = 0.31$ is reached.

Assume $\Delta^A = \Delta^B = 0.31$. Type Q is now a net buyer of intermediate shares. Q's net trade with B is much larger than his net trade with A. Because $q_0/C \approx 0.99936 < 1$, types A and B subsidize type Q. Due to the increase in cost and output, all types including Q are worse off at $t = 0$, and better off at $t > 0$, than in the partnership. In total, Q is the only type that prefers the corporation over the partnership. Furthermore, $CS_{y^*} \approx -8.1 \times 10^{-5}$ and $ES_{y^*} \approx -8.9 \times 10^{-5}$.

Consider $\Delta^A = \Delta^B = 0.4$. Output and cost rise again and $q_0/C \approx 0.9989$. A and B subsidize Q more than in the previous case, but qualitatively the picture remains the same. A and B prefer the partners0hip and Q prefers the corporation. Because $CS_{y^*} \approx -1.58 \times 10^{-4}$ and $ES_{y^*} \approx -1.85 \times 10^{-4}$, type Q gains more than A and B lose together so that the corporation is again socially preferred.

Finally, let $\Delta^A = \Delta^B$ be nearly equal to 0.5 so that Δ^Q is very close to 0. The picture becomes more pronounced but remains basically unchanged apart from the magnitudes of the effects. Now q_0/C is nearly equal to 1 and $CS_{y^*} \approx -4.3\times 10^{-4}$ and $ES_{y^*} \approx -2.7\times 10^{-4}$ and the corporation equilibrium is preferable.

5 Discussion of the example

In the example, the partnership equilibrium violates the first order condition for welfare maximization and minimal efficiency. One is led to ask questions of what makes the constraint $q_0(y_+) = C(y_+)$ binding and of what enables a corporation to mitigate the consequences of a binding constraint.

First, consider a weak planner who cannot change the production plan but who can assign the shares ϑ_0^A and ϑ_0^B and adjust shares of type Q accordingly. When this planner satisfies the same first order condition as the partnership then $\vartheta_0^A \approx 0.03047$ is smaller and $\vartheta_0^B \approx 0.02401$ is larger than in the partnership equilibrium. The move from the partnership equilibrium to the weak planner's equilibrium generates welfare and efficiency gains measured by $CS \approx ES \approx -0.00039$.

Second, consider a stronger planner who can also change the production plan. This planner increases the output strictly and the cost increases from 666.862 to 667.073 and the shares. In comparison to the partnership equilibrium, one obtains $CS \approx -0.00044$ and $ES \approx -0.00045$.

The corporation can partially imitate the second planner by selling its output below costs to increase its output, while the consumers continue to act as price takers who ignore the impact of their decisions on market clearing prices. However, a corporation can aim to improve welfare locally in a way that is not available to the partnership. This can be described as follows.

Consider a production plan y^* that gives rise to a *regular partnership equilibrium.* That is to say, $D\mathcal{W}_{y^*}(\hat{y}^*)$ has maximal rank $S-1$ because $\mathcal{Y}_{\text{part}}$ has codimension 1.

The *corporation equilibrium* induced by y^* cannot be regular when all initial shares are held by type Q because the total lack of an income effect prohibits any impact on $\mathcal{W}_{y^*}$. Hence, the corporation equilibrium at y^* cannot possess the full rank S in this particular case. That is to say, the corporation equilibrium must be critical.

Suppose that some initial shares are held by non-quasilinear consumers. Then the rank of $\mathcal{W}_{y^*}$ does not necessarily increase from $S-1$ to S because the effect can be of second order. This occurs when the equilibrium index, that is to say the sign of the Jacobian determinant of $D\mathcal{W}_{y^*}(y^*)$, changes its sign. Typically, however, the corporation equilibrium at y^* will be regular when initial shares are held by quasilinear consumers.

The following Proposition shows that the functional form of the example is irrelevant as long as the corporation equilibrium at y^* is regular. In the above example, this assumption is satisfied and the index equals 1.

The individual demand functions as well as the price functions q_s are assumed to be C^1 (non-vanishing Gaussian curvature). Spaces of C^1 functions are endowed with the topology of uniform C^1 convergence on compact subsets of their domain.

Proposition. Let $\mathcal{W}_{y_+^*}$ be of class C^2 in a neighborhood of y_+^*. Assume that the corporation equilibrium at y_+^* is regular. Then there are corporation equilibria with output vectors $\tilde{y}_+$ arbitrarily close to y_+^* such that $\mathcal{W}_{y^*}(\tilde{y}_+) > \mathcal{W}_{y^*}(y^*)$.

Proof. The regularity of the corporation equilibrium at y_+^* implies that there exists a neighborhood V of y_+^* and an $\epsilon > 0$ such that every C^1 function in the ϵ-neighborhood of $D\mathcal{W}_{y^*}$ maps V diffeomorphically onto some neighborhood of $0 \in \mathbb{R}^S$. The equation $D\mathcal{W}_{y^*}(y_+) = 0$ has a unique solution $\tilde{y}_+$ in V. At y_+^*, an infinitesimal move in the direction of the welfare gradient $D\mathcal{W}_{y^*}$ gives rise to a first order welfare gain. □

Another feature of the example is less robust. Whether Kaldor-Hicks comparisons lead to a clear conclusion depends on whether CS and ES have the same sign. In the quasilinear case, there is a unique, well-defined surplus concept because good 0 can be used to transfer utility so that CS and ES are identical. When one departs from the quasilinear case a second order effect drives the two surpluses apart.

In the example, fifty out of seventy consumers are of the quasilinear type Q. Furthermore, the allocation of initial shares across types A and B is symmetric. As a consequence, all Kaldor-Hicks comparisons that have been examined in the numerical calculations are conclusive.

Suppose now that the number of type C consumers is reduced to ten. Then the following happens. When $\delta^A = \delta^B$ is very small then $CS < 0$ and $ES > 0$. Furthermore, $ES > 0$ is closer to 0 than CS. This is still the case when $\delta^A = \delta^B = 0.01$. However, when $\delta^A = \delta^B$, has reached the level 0.02 then $ES < 0$. From there on, the picture stays qualitatively similar to the one described in Section 4.

6 Conclusions

A substantial part of the literature on production economies with incomplete markets follows Drèze (1974) and Magill and Quinzii (1996) and deals with economies in which all agents behave competitively. Every consumer maximizes utility given his state price system and every firm maximizes profits given its state price system. Drèze (1974) has shown that the first order condition for constrained efficiency in a two-period finance economy implies that a firm should maximize profits with respect to a convex sum of its shareholders' state price systems. However, when one leaves the framework of two-period finance models one encounters serious difficulties; see, for instance, Geanakoplos et al. (1986), Geanakoplos et al. (1990), and Magill and Quinzii (1996).

This paper considers multi-period models of finance economies with production and constant returns to scale and places the emphasis on the role of initial shares. It is no longer taken for granted that firms act competitively. The paper connects the literature on production economies with incomplete markets with the literature on Cournot-Walras models.

The paper illustrates a source of inefficiency that differs form those pointed out in the literature mentioned above. The inefficiency occurs because not only consumers but also firms act as price takers. These effects can be mitigated by corporations with initial shares. In a partnership economy with constant returns, the consumers pay in total the production costs C in order to obtain the dividend stream, whereas the initial shares of a corporation are sold at a price q_0 that can be below or above C.

Price taking agents are myopic in the sense that they ignore the impact of their portfolio choices on subsequent market prices. Therefore, the portfolio choices of consumers can lead to over- or underproduction. When there is underproduction as in the example in Section 4, a price $q_0 < C$ tends to stimulate the demand for shares so that the scale of production is increased due to income effects. A partnership is unable to make such a correction. As shown in the example, *corporation equilibria can Pareto dominate partnership equilibria* if the initial shares are suitably chosen.

However, Pareto comparisons cannot be made for most allocations of initial shares. Therefore, the paper uses a Kaldor-Hicks compensation criterion that involves two Hicksian surplus concepts, the compensating surplus and the equivalent surplus. The compensating surplus measures losses due to inefficiency, the equivalent surplus measures welfare changes. Only two equilibria at a time are compared so that the lack of transitivity of Kaldor-Hicks comparisons is irrelevant.

The role of the initial shares is examined for a variety of initial allocations where the Kaldor-Hicks comparison is conclusive. The computations show that *selling shares below costs can improve welfare as well as efficiency* also when the equilibria cannot be Pareto ranked. In other examples, the situation is similar when corporations sell their shares above cost. It can happen that the compensating and the equivalent surplus have different signs and do not lead to a clear conclusion.

References

Dierker, E., H. Dierker, and B. Grodal: Are incomplete markets able to achieve minimal efficiency? *Economic Theory* **25**, 75–87 (2005)

Dierker, E. and H. Dierker: Drèze equilibria and welfare maxima. *Economic Theory* **45**, 55–63 (2010a)

Dierker, E. and H. Dierker: Welfare and efficiency in incomplete markets with a single firm. *J. Math. Econ.* **46**, 652–665 (2010b)

Dierker, E.: A multiperiod Drèze rule. *Economic Theory Bulletin* **3**, 129–151 (2015)

Drèze, J.H., Investment under private ownership: Optimality, equilibrium and stability. In: Drèze J.H. (ed.) Allocation Under Uncertainty; Equilibrium and Optimality. New York: Wiley, 129–165 (1974)

Gabszewicz, J and J.P. Vial., Oligopoly "à la Cournot" in a general equilibrium analysis. *Journal of Economic Theory* **4**, 381–400 (1972)

Geanakoplos, J. and H. Polemarchakis: Existence, regularity, and constrained suboptimality of competitive allocations when markets are incomplete. In: Uncertainty, Information, and Communication: Essays in Honor of Kenneth Arrow, vol. 3, W.P. Heller, R.M. Ross, and D.A. Starett eds., Cambridge: Cambridge University Press (1986)

Geanakoplos, J., M. Magill, M. Quinzii, and J. Drèze: Generic inefficiency of stock market equilibrium when markets are incomplete. *Journal of Mathematical Economics* **19**, 113–151 (1990)

Grossman, S. and O. Hart, A Theory of Competitive Equilibrium in Stock Market Economies, *Econometrica* **47**, 293-330 (1979)

Guesnerie, R.: Pareto optimality in non-convex economies. Econometrica **43**, 1–31 (1975)

Hicks, J.: A Revision of Demand Theory, Oxford: Oxford University Press (1956)
A Course in Microeconomic Theory, Hemel Hempstead: Harvester Wheatsheaf (1990)

Magill, M. and M. Quinzii: Theory of Incomplete Markets. Cambridge, London: MIT Press (1996)